

ONE-CLASS PEELING FOR OUTLIER DETECTION IN HIGH DIMENSIONS

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EXAMPLE

Statistical Learning Methods Applied to Process Monitoring: An Overview and Perspective

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The increasing availability of high-volume, high-velocity data sets, often containing variables of different data types, brings an increasing need for monitoring tools that are designed to handle these big data sets. While the research on multivariate statistical process monitoring tools is vast, the application of these tools for big data sets has received less attention. In this expository paper, we give an overview of the current state of data-driven multivariate statistical process monitoring methodology. We highlight some of the main directions involving statistical learning and dimension reduction techniques applied to control charts in research from supply chain, engineering, computer science, and statistics. The goal of this paper is to bring into better focus some of the monitoring and surveillance methodology informed by data mining techniques that show promise for monitoring large and diverse data sets. We introduce an example using Wikipedia search information and illustrate a few of the complexities of applying the available methods to a high-dimensional monitoring scenario. Throughout, we offer advice to practitioners and some suggestions for future research in this emerging area of research.

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The **National Football League (NFL)** is concerned with monitoring “public relations events”, or at least separate out the typical traffic from an “event”.

`http://dumps.wikimedia.org/other/pagecounts-raw`

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Every hour contains a compressed file of approximately 100MB for the number of hits on millions of Wikipedia pages. A **week of data** holds **over 16GB** of storage.

Our sample considers **page hits per hour** over a two week period for all **NFL active players, coaches, managers and teams** beginning on **9/1/2014** and was specifically chosen to include the first two weeks of the 2014 season for a total of **n=354** samples.

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The number of pages for teams, currently active players, coaches, and managers is **p=1917**.

Early Lead

The details of Adrian Peterson's arrest are disturbing

By **Des Bieler** September 12, 2014 [✉ Email the author](#)

A warrant has been issued in Montgomery County, Texas, for the arrest of Vikings running back Adrian Peterson, after he was indicted on a felony charge for reckless or negligent injury to a child. [A report by CBS Houston](#) has some details, and they are disturbing.

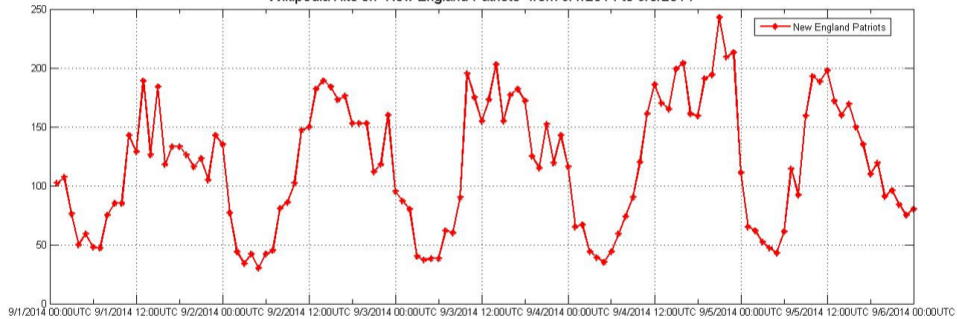
WARNING: VIOLENCE

Ray Rice Punch Video Released In Full By TMZ

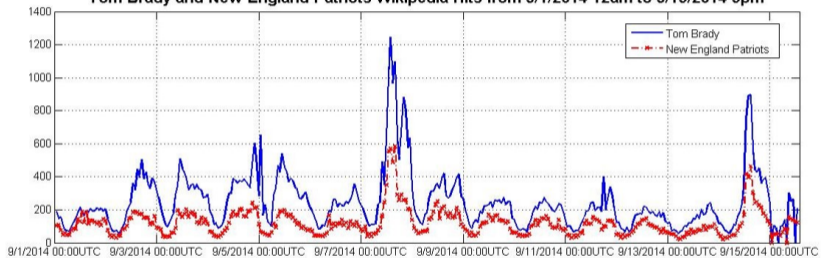
47 diggs [TMZ](#) [Sports](#)

Ray Rice was originally suspended for just two games — though it was later extended — after a previous video leaked showing him dragging his unconscious fiancée out of an elevator. Now we know exactly what happened inside that same elevator.

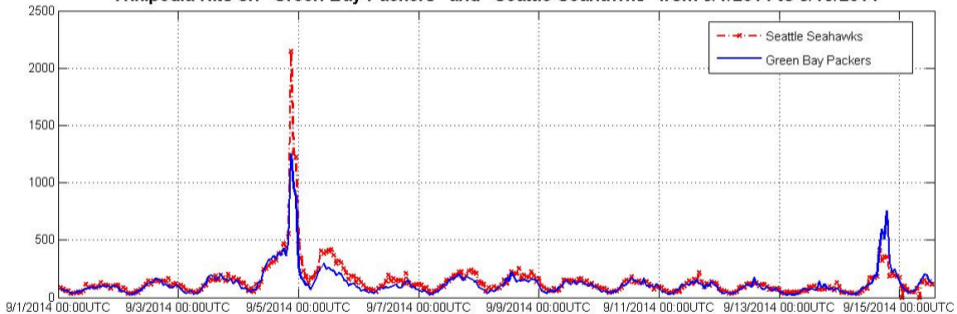
Wikipedia Hits on "New England Patriots" from 9/1/2014 to 9/5/2014



Tom Brady and New England Patriots Wikipedia Hits from 9/1/2014 12am to 9/15/2014 5pm



Wikipedia Hits on "Green Bay Packers" and "Seattle Seahawks" from 9/1/2014 to 9/15/2014



EVENT IDENTIFICATION

1. Searched "NFL" on Google news in a custom search for a custom date range by day (based on PDT converted to EDT) during the two week period.
2. We only considered news stories that appeared on the first page of the search results (approximately 10 search returns).
3. If a news story was new to any period we consider it to be breaking news. If it was classified as breaking news then we went to the source of the news which first reported the story to find the exact time that the story was released and converted it to EDT.
4. If the story drops during the overnight hours 1am and 8am we start our period at 8am EDT the morning following.
5. If the story drops between 6am and 1am EDT then we consider a potential signal starting within a 5 hour lag of the time of the story hitting.
6. NFL games will signal based on the de-trending method.

1. A method that does not need to be stopped and re-calibrated once a signal is observed.
2. We need a method that can handle count data (**not multivariate normal**), contains many zero values, and has a nested correlation structure.
3. A method that can be **adapted** to data that occurs over time.

METHODS

1. Statistical Distance Methods.
2. k-nearest neighbor (kNN) methods.
3. Density Estimation Methods.

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3. Density Estimation Methods.

Most multivariate outlier detection methods follow two steps:

1. Robust estimation of the center and scale of the data.
2. Evaluation of a measure of “outlyingness”, i.e. a distance measure.

The following three methods use these two steps and can be applied when $p > n$.

Outlier detection for high-dimensional data

BY KWANGIL RO, CHANGLIANG ZOU, ZHAOJUN WANG

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SUMMARY

Outlier detection is an integral component of statistical modelling and estimation. For high-dimensional data, classical methods based on the Mahalanobis distance are usually not applicable. We propose an outlier detection procedure that replaces the classical minimum covariance determinant estimator with a high-breakdown minimum diagonal product estimator. The cut-off value is obtained from the asymptotic distribution of the distance, which enables us to control the Type I error and deliver robust outlier detection. Simulation studies show that the proposed method behaves well for high-dimensional data.

Some key words: Masking; Minimum covariance determinant estimator; Reweighting; Swamping.

Ro et al. (2015) use a modified Mahalanobis (see equation 1) distance that uses only the **diagonal elements** of the sample **covariance matrix**.

$$d_i^2(\mu, D) = (Y_i - \mu)^T D^{-1} (Y_i - \mu) \quad (1)$$

Where D and μ are estimated from a subset of observations such that determinant of the diagonal elements of the covariance for that subset of observations is minimal.

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Outliers are determined by **setting a significance level** α , defining a rejection region. Points with **distances in the rejection region** are flagged.



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Outlier identification in high dimensions

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Abstract

A computationally fast procedure for identifying outliers is presented that is particularly effective in high dimensions. This algorithm utilizes simple properties of principal components to identify outliers in the transformed space, leading to significant computational advantages for high-dimensional data. This approach requires considerably less computational time than existing methods for outlier detection, and is suitable for use on very large data sets. It is also capable of analyzing the data situation commonly found in certain biological applications in which the number of dimensions is several orders of magnitude larger than the number of observations. The performance of this method is illustrated on real and simulated data with dimension ranging in the thousands. © 2007 Elsevier B.V. All rights reserved.

Keywords: Outlier identification; Robust estimators; High dimension; Robust principal components

Filmozer et al. (2008) first reduce the dimension using PCA.

1. Run PCA and keep PC's to explain 99% of the original variation.
2. Location outliers are ranked on what is equivalent to a robust Mahalanobis distance (w_{1i}).
3. Scale outliers are ranked determined based on a modified bi-weight function (w_{2i}).
4. Each data point is then assigned a weight according to:

$$w_i = \frac{(w_{1i} + s)(w_{2i} + s)}{(1 + s)^2}$$

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Outliers are classified as **those observations that have $w_i < 0.25$** with $s = 0.25$

OCP METHOD

The One-Class Peeling (OCP) method uses Support Vector Data Description (SVDD) to **peel away outlying** observations, similar to Convex Hull Peeling.

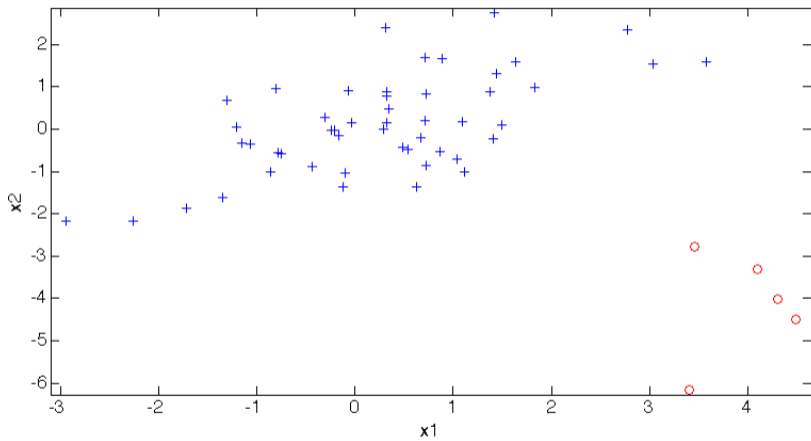
The mean estimate, $\hat{\mu}_{\text{OCP}}$, is the mean of the last two observations remaining after the others are peeled.

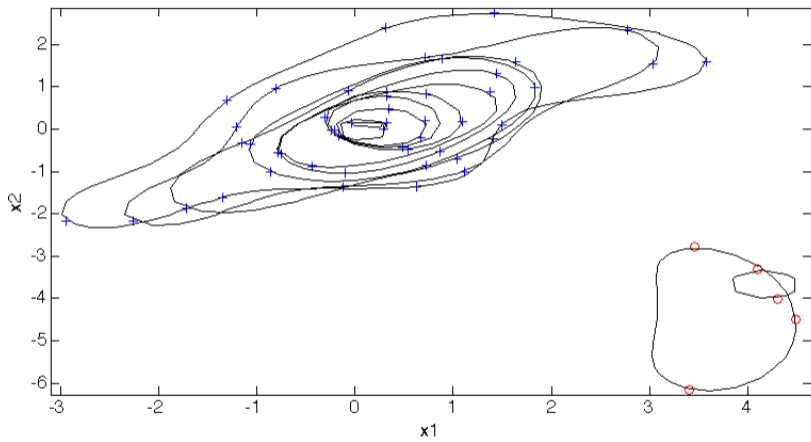
Distances are relative to the value of $\hat{\mu}_{\text{OCP}}$.

- SVDD creates a **hyper-sphere boundary** around a sample of data.
- The boundary is created using only a few points called **support vectors**.
- The user can specify how **tight** the boundary fits the data.
- If the data are mapped to the kernel space then SVDD can create a **flexible boundary** around a sample of data.
- A common choice is the Gaussian kernel:

$$KS_G(\mathbf{x}_i, \mathbf{x}_j) = \exp\left(-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{s^2}\right) \quad (2)$$

- We set $s = p$ based on Weese et al. (2016).





A breakdown point describes the least percentage of outliers in a set S at which the estimate becomes arbitrarily incorrect.

We show empirically that the **breakdown point of the OCP method is between 30% and 35% contamination** for correlated and uncorrelated normal and t ($df=10$) distributed data for dimensions of $p = 25$ to $p = 100$.

The second step of the OCP method calculates a distance measure from each observation to the estimated mean, $\hat{\mu}_{\text{OCP}}$.

We use the Gaussian kernel to retain the same feature space that was used to develop the SVDD boundaries.

Recall, a kernel function is also a similarity function.

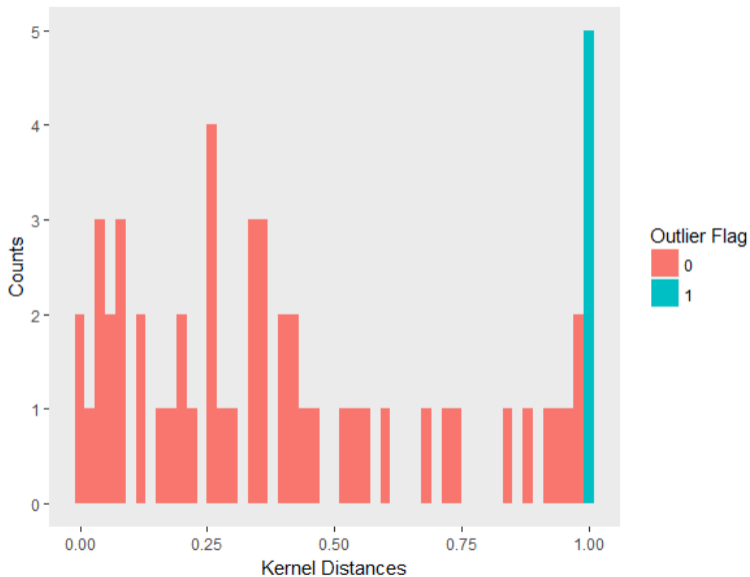
The Gaussian kernel similarity (Equation 2) is a decreasing function of the Euclidean distance between two arbitrary points and $0 \leq \text{KS}_G(\mathbf{x}_i, \mathbf{x}_j) \leq 1$.

We use the following linear transformation of Equation 2 to form our distance metric:

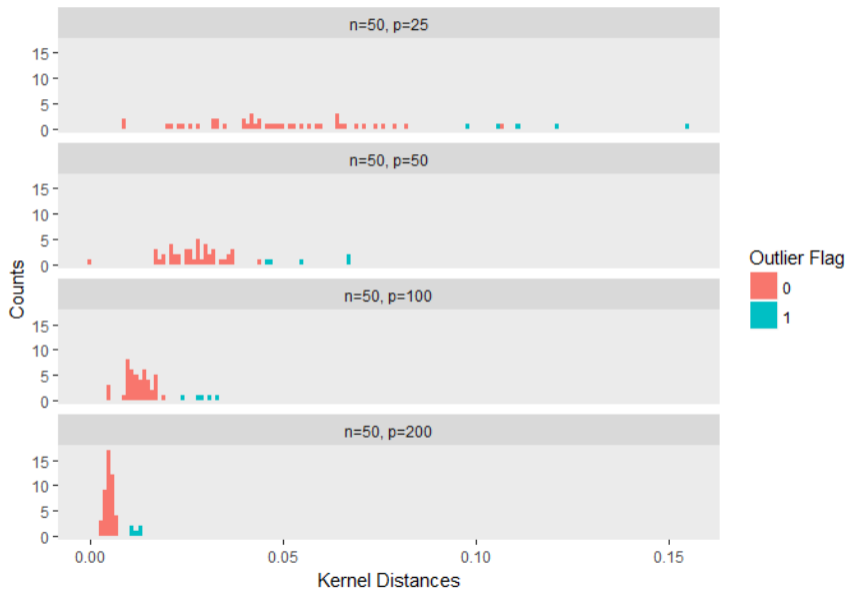
$$\text{KD}_{\text{OCP}}(\mathbf{x}_i, \hat{\boldsymbol{\mu}}_{\text{OCP}}) = 1 - \exp\left(-\frac{\|\mathbf{x}_i - \hat{\boldsymbol{\mu}}_{\text{OCP}}\|^2}{s^2}\right) \quad (3)$$

Smaller values of $\text{KD}_{\text{OCP}}(\mathbf{x}_i, \hat{\boldsymbol{\mu}}_{\text{OCP}})$ (closer to 0) indicate observations close to the estimated mean $\hat{\boldsymbol{\mu}}_{\text{OCP}}$, while larger values of $\text{KD}_{\text{OCP}}(\mathbf{x}_i, \hat{\boldsymbol{\mu}}_{\text{OCP}})$ (closer to 1) indicate observations far away from $\hat{\boldsymbol{\mu}}_{\text{OCP}}$.

Kernel distances from $n=50$, $p=2$, $N(0,1)$ Observations



Kernel Distances for $N(0,1)$ generated datasets



Unfortunately the values of the KD_{OCP} (Equation 3) change with distribution and sample size.

So we scale the KD_{OCP} by

$$sRKD = \frac{KD_{OCP} - \text{median}(KD_{OCP})}{sMAD(KD_{OCP})}$$

where the $sMAD = b * \text{med}_i |x_i - \text{med}_j x_j|$ and $b = \frac{1}{\sqrt{2}} * \Phi^{-1}(ct/2)$ and where Φ^{-1} is the inverse complementary error function.

For any distribution, an $sRKD > 4$ is flagged as an outlier using factor $b = 1.3238$ for sMAD.

This controls the empirical Type I error rate to a maximum 5% for data of any distribution.

But of course a user can change the threshold at which they wish to flag potential outliers.

WHAT ABOUT CHANGES THROUGH TIME?

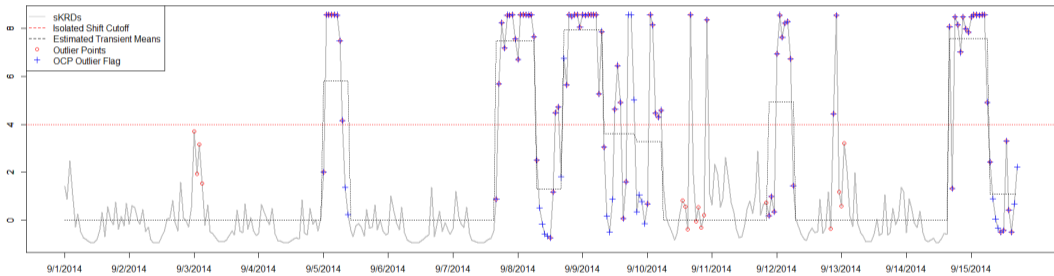
Since the sRKD values are univariate, there are many options.

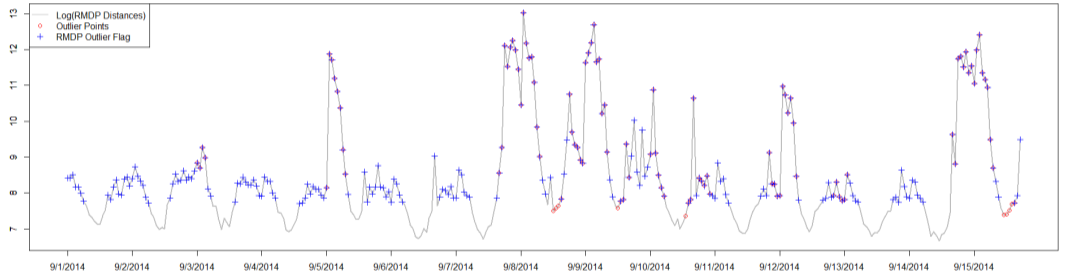
We choose to use the RSP change point method of Capizzi and Masarotto (2017).

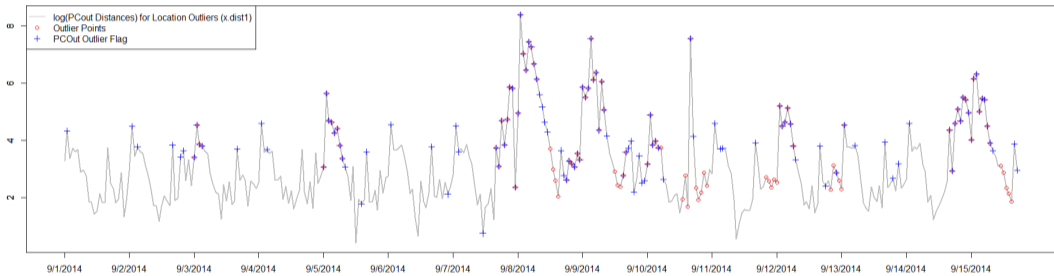
This method will signal if the mean of the sRKDs has shifted during the time period.

We choose $\alpha = 0.05$ for the RSP method.

EXAMPLE RESULTS







Method	Detection Rate (%)	False Positive Rate (%)
OCP	84.7	9.2
R-MDP	91.4	56.6
PCOut	71.4	19.2

*Recall, events were labeled according to the protocol on slide 12.

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A FEW SIMULATIONS

We simulated several different sample sizes and dimensions with 0, 5, 10, 20 and 30 percent outliers.

We did this for data sampled from correlated and uncorrelated Normal, correlated Lognormal and correlated $t(df=10)$.

To maintain equivalent shifts across distributions we shifted the outlying observations probabilistically. We shifted in random directions.

The following results are for sustained outlier shifts, i.e. a step change starting at a random time. Results for isolated and transient shifts are similar.

SIMULATION RESULTS

		0% Outliers		
		OCP	R-MDP	PCOut
		Total Error	Total Error	Total Error
Normal	N = 50, p = 100	0.30%	22.83%	10.02%
Uncorrelated	N = 100, p = 100	0.26%	12.61%	7.99%
Normal	N = 50, p = 100	0.44%	20.70%	10.02%
Correlated	N = 100, p = 100	0.39%	12.10%	9.01%
T, df=10	N = 50, p = 100	4.73%	38.79%	14.45%
	N = 100, p = 100	5.14%	41.51%	17.91%
Lognormal	N = 50, p = 100	6.03%	49.20%	20.82%
	N = 100, p = 100	6.55%	52.28%	23.31%

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SIMULATION RESULTS

10% Outliers

		OCP		R-MDP		PCOut	
		Detection	Total Error	Detection	Total Error	Detection	Total Error
Normal	N = 50, p = 100	100.00%	0.02%	99.84%	18.15%	100.00%	3.63%
Uncorrelated	N = 100, p = 100	100.00%	0.03%	100.00%	10.24%	100.00%	3.26%
Normal	N = 50, p = 100	99.90%	0.13%	99.80%	14.20%	99.98%	3.92%
Correlated	N = 100, p = 100	100.00%	0.08%	100.00%	9.17%	100.00%	3.59%
T, df=10	N = 50, p = 100	100.00%	2.01%	100.00%	32.22%	100.00%	9.26%
	N = 100, p = 100	100.00%	1.84%	100.00%	35.19%	100.00%	12.98%
Lognormal	N = 50, p = 100	96.66%	7.20%	99.30%	39.94%	98.96%	15.99%
	N = 100, p = 100	98.67%	5.78%	99.37%	43.37%	99.36%	19.09%

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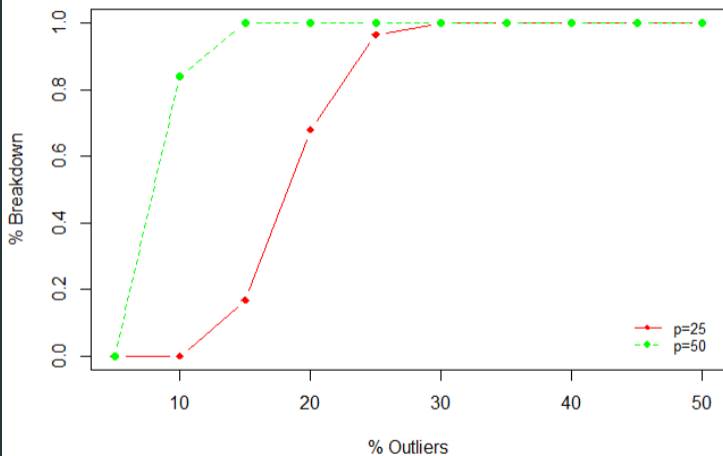
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- The OCP method out performs existing methods in terms of total error rate using a realistic data stream application.
- The OCP method out performs existing methods in terms of total error rate using simulation data.
- In general, using the OCP method will lead to a larger baseline sample compared to R-MDP and PCOut.

QUESTIONS?

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BACKUP

CHP Breakdown, Uncorrelated Normal



OCP Breakdown, Uncorrelated Normal

