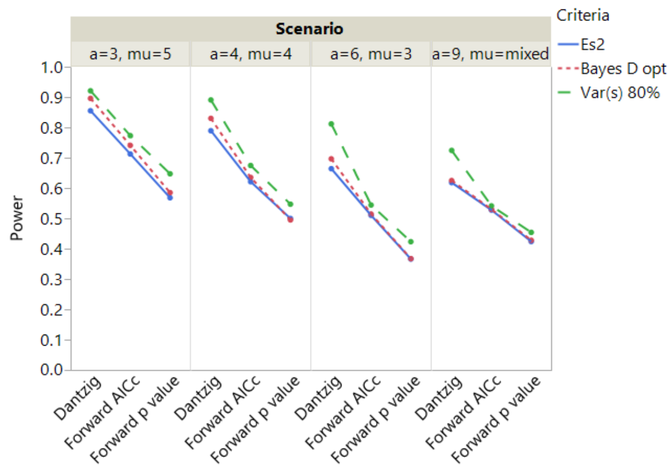
Supplementary Material C: Additional Simulations for "Powerful Supersaturated Designs when Effect Directions are Known"

C1 Introduction

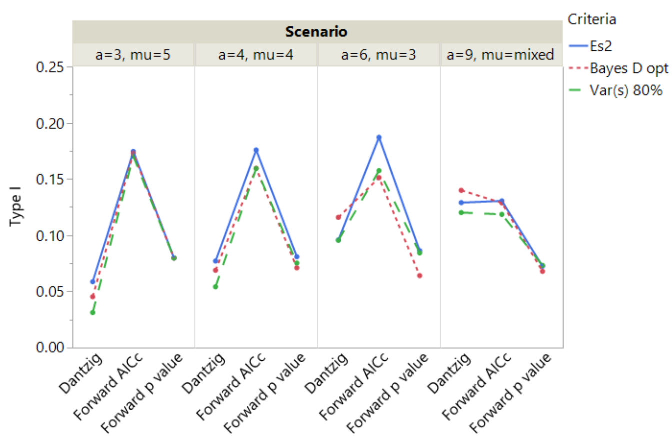
The simulation protocol described in Section 4.2 of the main paper assigns a small positive value (i.e., from $abs(N(0, 0.2))$) to the inactive factors. Furthermore, as with the active factor columns, the signs of the coefficients of the inactive factors are permitted to change sign with probability $p \in \{0, 0.25, 0.5\}$. In Sections 4.4 and 4.5, we provide empirical evidence that the constrained positive $Var(s)$-optimal designs have significantly better power when up to 25% of the signs of coefficients are misspecified and power that is no worse than the other design types when half of the signs are misspecified. In addition to these results, we have also run simulations with the same protocol as described in Section 4.2 except that inactive columns are assigned a true coefficient of 0. In this case, when p is varied, only the *active* factors change signs. In the following sections, we show that even under this alternative experimental condition, the constrained positive $Var(s)$-optimal designs are dominant with regards to power when most of the signs are specified correctly and no worse than other criteria when sign misspecification occurs.

C2 Simulation Results when Effect Directions are Known ($p = 0$)

Figure 1 presents simulation results for Power and Type I error rate when $p = 0$ (i.e., none of the active effect directions are misspecified). Figure 1a shows the overall dominance of the $Var(s)$ criteria and Figure 1b shows that $Var(s)$ designs do not produce differences in Type I error rate when compared to the more traditional criteria.



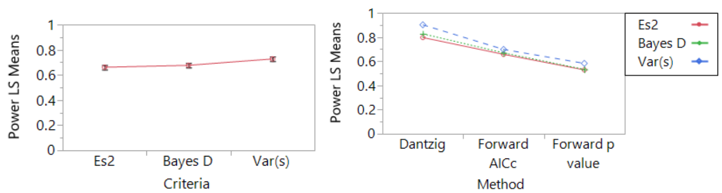
(a) Power



(b) Type 1 Error

Figure 1: Average power, type 1 error, and coverage based on simulations where inactive factors are assigned a true coefficient value of 0. Results shown for $p = 0$ over all twenty designs sizes, shown for each of the three analysis methods and each of the four scenarios.

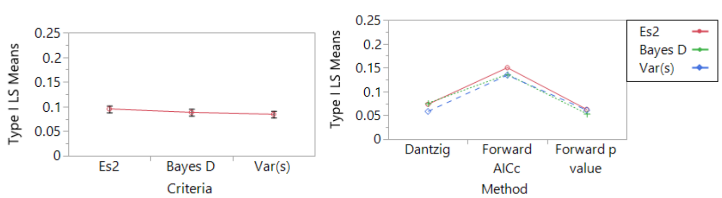
A two factor interaction model was fit to the square root of the counts of correctly identified effects when $p = 0$ using n, k Criteria, Method, and Scenario as factors ($R^2 \approx$ 92%). Figure 2 shows the main effect plot of Criteria (p-value < 0.0001) as well as the interaction plot between Criteria and Method. A Tukey multiple comparison procedure indicates that the constrained positive $Var(s)$-optimal designs have the highest overall power when compared to the other criteria. Regarding the interaction between criteria and method, a Tukey multiple comparison test indicates that the $Var(s)$ designs analyzed with the Dantzig selector have the highest overall power and is significantly different from all other method and criterion combinations.



(a) Criteria Main Effect Plot (b) Criteria*Method Interaction

Figure 2: Criteria main effect and interaction plot of power when $p = 0$. Note the y-axis is adjusted to show proportions.

A similar formal analysis of the Type I error rate counts ($R^2 \approx 93\%$) shows Criteria to be significant (p-value = 0.0061) and a Tukey analysis shows that $E(s^2)$-optimal designs have the highest overall Type I error rate and is significantly higher than the constrained positive $Var(s)$ designs. The Criteria by Method interaction is also significant (p-value = 0.0011) and a Tukey analysis shows that all three criteria analyzed by Forward Selection using AICc have significantly higher Type I errors than all other method and criteria combination. See Figures 3a and 3b for the main effect and interaction plots, respectively.



(a) Criteria Main Effect Plot (b) Criteria*Method Interaction

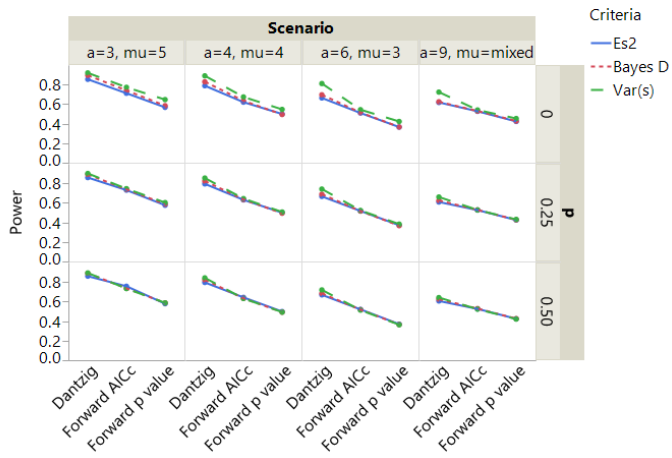
Figure 3: Criteria main effect and interaction plot of Type I error when $p = 0$. Note the y-axis is adjusted to show proportions.

C3 Effect of Misspecifying Effect Directions

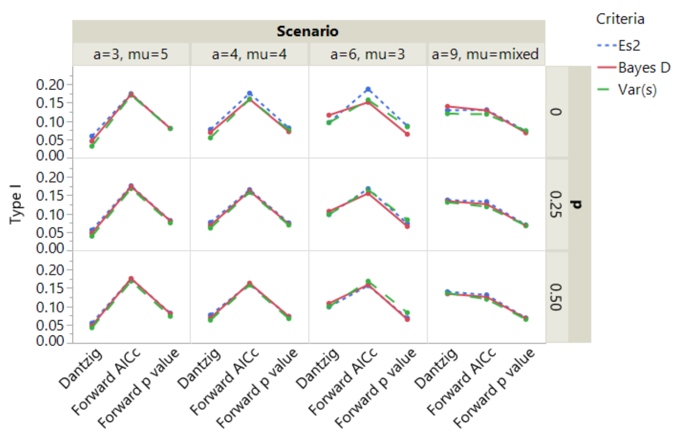
When the signs of only the active effects are positive, we have shown that the constrained positive $Var(s)$-optimal designs have a higher power versus traditional design construction criteria (along with similar type I error rates). In this section we explore the consequence of incorrect specification of the direction of the *active* effects. As in section 4.5 of the main paper, we perform simulations in which we systematically vary the proportion of misspecified effects. Figure 4 shows the average power and Type I error for each of the criteria, for each value of p and each scenario across all designs sizes. When comparing the rows of 4a, we notice that the advantage of the constrained positive $Var(s)$ designs are most pronounced for $p = 0$, slightly visible for the Dantzig selector for $p = 0.25$ and disappear for $p = 0.5$.

A two-factor interaction model was fit to the simulation data depicted in Figure 4a, where again we take as the response the square root of the counts of correctly identified effects ($R^2 \approx 92\%$). We use as factors p, Criteria, Scenario, Method, n, and k. Figure 5 shows the main effect plot of Criteria (p-value< 0.0001) as well as the interaction plot between Criteria and p (p-value $= 0.0003$). These results confirms what Figure 4a suggested: when the effect signs are correctly or mostly correctly specified the constrained positive $Var(s)$ designs are more powerful, but even when signs are misspecified the constrained positive $Var(s)$ designs are no worse than standard designs (Figure 5b).

We believe the results of these additional simulations, where only the active factor coefficient directions are allowed to vary, strengthen the conclusions of the results displayed in Section 4 of the main paper. We have shown that even when the bias is solely due to active factors, the constrained positive $Var(s)$ designs remain dominant in terms of power when the signs of the factors are specified correctly and are no worse when the signs are misspecified.

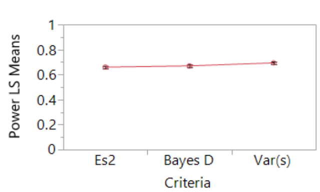


(a) Power

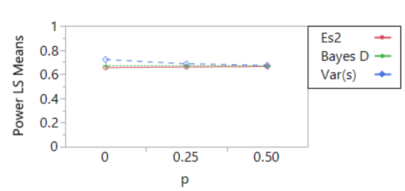


(b) Type 1 Error

Figure 4: Average power and type 1 error by design criteria over all twenty designs sizes, shown for each of the three analysis methods, every value of p, and each of the four scenarios.



(a) Criteria Main Effect Plot



(b) Criteria*p Interaction

Figure 5: Criteria main effect and interaction plot of power over all values of p. Note the y-axis is adjusted to show proportions.